Non-empty beliefs: belief revision using BSML

Tomasz Klochowicz
ILLC, University of Amsterdam

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Belief revision

Theories of belief revision describe how agents change their beliefs in light of new information (Alchourrón et al., 1985).

Dispositions to believe are formally represented as: $\mathrm{B}^{\varphi}\psi$, which can be read as 'after learning/being informed that φ, the agent would believe ψ'.

For instance, 'after learning $p \wedge q$ the agent would believe that p' is a tautology in most frameworks: $\models \mathrm{B}^{p \wedge q} p$.

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Alchourrón et al. (1985) (AGM) propose that agents' belief sets are closed under classical logical consequence: If $\varphi \models \psi$ then $\models \text{B}^{\varphi}\psi$.

Believing disjunctions

Disjunctive assertions convey uncertainty (Grice, 1991; Aloni, 2022):

(1) Ann has two or three children. $\alpha \vee \beta$
$\rightsquigarrow$ Possibly Ann has two children and possibily three. $\Diamond\alpha \wedge \Diamond\beta$

(2) #I have two or three children.

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(2) #I have two or three children.

Disjunctive belief ascriptions convey uncertainty of the speaker or the subject:

(3) a. #Ann believes that she has two or three children. $\text{B}(\alpha \vee \beta)$
b. Ann believes that she has two or three children. $\text{B}\alpha \vee \text{B}\beta$

Disjunction in belief revision

Beliefs are not closed under disjunction introduction:

(4) Ann believes that she has two children $\quad$ Bα
$\not\rightsquigarrow$ Ann believes that she has two or three children. $\quad$ B$(\alpha \vee \beta)$

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(4) Ann believes that she has two children $\text{B}\alpha$
$\not\rightsquigarrow$ Ann believes that she has two or three children. $\text{B}(\alpha \vee \beta)$

Belief revision is not closed under disjunction introduction:

(5) # After learning that 'Shakespeare wrote Hamlet' (p), my student would believe that 'Shakespeare or Dickens wrote Hamlet' ($p \vee q$). (Hansson, 2022)

Thus, belief sets and belief revision are not closed under classical logic.

Non-classical belief closure

We exhibit rationality in our beliefs (at least I hope so). Hence, many argue for a non-classical logic as a closure for belief sets.

(see e.g., Jago, 2014; Berto, 2019; Berto and Nolan, 2023; Hansson, 2022)

Most approaches follow the slogan: "thought is hyperintensional": the equivalence relation under a belief operator is more fine-grained than 'denoting the same set of possible worlds':

- $\varphi \equiv \psi \not\models \mathsf{B}\varphi \not\equiv \mathsf{B}\psi$
- $\varphi \equiv \psi \not\models \mathsf{B}^{\varphi}\chi \not\equiv \mathsf{B}^{\psi}\chi$

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Hence:

$$\varphi \equiv \psi \text{ iff } |\varphi| = |\psi|$$

$$\mathsf{B}^{\varphi}\chi \equiv \mathsf{B}^{\psi}\chi \text{ iff } |\varphi| = |\psi| \text{ and } c(\varphi) = c(\psi)$$

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Hence:

$$\varphi \equiv \psi \text{ iff } |\varphi| = |\psi|$$

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In particular:
$|p| \subseteq |p \vee q|$ but $c(p) < c(p \vee q)$ hence $p \models p \vee q$ but $\not\models \mathsf{B}^{p}(p \vee q)$

Predicted inference patterns

(Success) $\models \mathrm{B}^{\varphi}\varphi$

(Simplif) $\mathrm{B}^{\varphi}(\psi \wedge \chi) \models \mathrm{B}^{\varphi}\psi$

(Adjunction) $\{\mathrm{B}^{\varphi}\psi, \mathrm{B}^{\varphi}\chi\} \models \mathrm{B}^{\varphi}(\psi \wedge \chi)$

(Non-monotonicity) $\mathrm{B}^{\varphi}\psi \not\models \mathrm{B}^{\varphi \wedge \chi}\psi$

(Disjunction 1) $\mathrm{B}^{\varphi}\psi \not\models \mathrm{B}^{\varphi}(\psi \vee \chi)$

(Disjunction 2) $\mathrm{B}^{\varphi}(\psi \vee \chi) \not\models \mathrm{B}^{\varphi}\psi$

(Cond) $\varphi \rightarrow \psi \not\models \mathrm{B}^{\varphi}\psi$

(R weakening) $\{\mathrm{B}^{\varphi}\psi, \psi \rightarrow \chi\} \not\models \mathrm{B}^{\varphi}\chi$

(CBI) $\{\mathrm{B}^{\varphi}\psi, \mathrm{B}^{\varphi}(\psi \rightarrow \chi)\} \not\models \mathrm{B}^{\varphi}\chi$

(LLE/Hyper) $\{\mathrm{B}^{\varphi}\chi, \varphi \equiv \psi\} \not\models \mathrm{B}^{\psi}\chi$

(Incons) $\not\models \mathrm{B}^{\varphi \wedge \neg\varphi}\psi$

(Taut) $\not\models \mathrm{B}^{\varphi}(\psi \rightarrow \psi)$ and $\not\models \mathrm{B}^{\varphi}(\psi \vee \neg\psi)$

(PEP) $\{\mathrm{B}^{\varphi}\psi, \mathrm{B}^{\psi}\varphi, \mathrm{B}^{\varphi}\chi\} \models \mathrm{B}^{\psi}\chi$

(CUT) $\{\mathrm{B}^{\varphi}\psi, \mathrm{B}^{\varphi \wedge \psi}\chi\} \models \mathrm{B}^{\varphi}\chi$

(CM) $\{\mathrm{B}^{\varphi}\psi, \mathrm{B}^{\varphi}\chi\} \models \mathrm{B}^{\varphi \wedge \psi}\chi$

Problematic inferences

However, the topic-oriented framework will have trouble accounting for the following inferences:

$$\text{(Poss) } \textsc{b}^{\varphi}(\alpha \vee \beta) \models \langle \textsc{b} \rangle^{\varphi} \alpha$$

$$\text{(Taut2) } \not\models \textsc{b}^{\varphi}(\varphi \vee \neg\varphi)$$

$$\text{(BSDA) } \textsc{b}^{\varphi \vee \psi} \alpha \models \textsc{b}^{\varphi} \alpha^{1}$$

[1]Where α is a formula without disjunction and negated conjunctions.

Problematic inferences

(Poss) $\mathsf{B}^{\varphi}(\alpha \vee \beta) \models \langle\mathsf{B}\rangle^{\varphi}\alpha$ (Taut2) $\not\models \mathsf{B}^{\varphi}(\varphi \vee \neg\varphi)$ (BSDA) $\mathsf{B}^{\varphi\vee\psi}\alpha \models \mathsf{B}^{\varphi}\alpha$

Since disjunctions convey uncertainty (possibility of each disjunct), then acquiring a disjunctive belief should lead to uncertainty.

Disjunctive beliefs should be strictly less informative than beliefs in one of the disjuncts.

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Disjunctive beliefs should be strictly less informative than beliefs in one of the disjuncts.

(5) #After learning that 'Shakespeare, not Dickens wrote Hamlet' $(p \wedge \neg q)$, my student would believe that 'Shakespeare or Dickens wrote Hamlet' $(p \vee q)$.

This example is problematic not because something 'off-topic' was introduced, but because we feel that the student considers it possible that not Shakespeare but Dickens wrote Hamlet; they learned a less informative statement.

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Since $c(p) = c(\neg p) = c(p \vee \neg p)$, $\mathsf{B}^{\varphi}(\varphi \vee \neg\varphi)$ is a validity in Berto (2019)'s system.

But if disjunctive belief ascriptions convey uncertainty of the subject, then clearly learning that it is raining should not automatically make the agent believe that it is possibly not raining.

Acquiring disjunctive information

Imagine that you believe that Bill is in his office, but you overhear a conversation between your coworkers:

(6) A: Where is Bill?
B: At the cafeteria. It is lunchtime.

Two possible reactions: 1. Ignore it since you certainly believe that Bill is in his office. 2. Accept that he is in the cafeteria.

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(7) A: Where is Bill?
B: Either in his office or at the cafeteria. It is lunchtime.

If disjunction has possibility implications, then the update should preserve them; learning disjunctions we learn not that 'at least one of the two options is true', but also that 'both options are possible'.

Problematic inferences

(Poss) $\text{B}^{\varphi}(\alpha \vee \beta) \models \langle \text{B} \rangle^{\varphi} \alpha$ (Taut2) $\not\models \text{B}^{\varphi}(\varphi \vee \neg\varphi)$ **(BSDA)** $\text{B}^{\varphi \vee \psi} \alpha \models \text{B}^{\varphi} \alpha$

Intuitions from the literature on counterfactuals (Alonso-Ovalle, 2004)

(8) If it rained or snowed, the party would be cancelled. $(r \vee s) > p$
$\rightsquigarrow$ If it rained the party would be cancelled. $r > p$

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(9) After learning that it rained or snowed ($r \vee s$), John believes that the party was cancelled (p) $\text{B}^{r \vee s} p$
$\rightsquigarrow$ If John learned that it rained he would believe that the party would have been cancelled. $\text{B}^{r} p$

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(10) *Context:* John learned that it is raining, so he is certain that the party will be cancelled. $\text{B}^{r} p$

a. #After learning that it will rain or not rain ($r \vee \neg r$), John is certain that the party will be cancelled (r). $\text{B}^{r \vee \neg r} p$

Clearly, in this context, the update is non-trivial as it introduces another possibility: not raining. Hence, John can no longer be certain that the party will be cancelled.

The logic of information states

Information states

- An information state is a set of possible worlds.
- Agent's belief set can be represented as an information state (e.g., the image of an accessibility relation). (Hintikka, 1962).
- Belief revision can be represented as a change from one information state to another.

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Let s be an information state. Classically:

$M, s \models \varphi$ iff $\forall w \in s$: $M, w \models \varphi$ iff $M, w \models_{S5} \Box\varphi$.

But consider:

If $M, w \models \neg p \land q$ then $M, \{w\} \models p \lor q$ even thought $M, \{w\} \models \neg\Diamond p$.

Aloni (2022)'s logic of information states

BSML clauses define logic equivalent to classical modal logic:

$M, s \models p$ iff $\forall w \in s : V(w,p) = 1$
$M, s \dashv p$ iff $\forall w \in s : V(w,p) = 0$

$M, s \models \neg\varphi$ iff $M, s \dashv \varphi$.
$M, s \dashv \neg\varphi$ iff $M, s \models \varphi$.

$M, s \models \varphi \vee \psi$ iff $\exists t, t' : t \cup t' = s \,\&\, M, t \models \varphi \,\&\, M, t' \models \psi$.
$M, s \dashv \varphi \vee \psi$ iff $M, s \dashv \varphi$ and $M, s \dashv \psi$.

$M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$.
$M, s \dashv \varphi \wedge \psi$ iff $\exists t, t' : t \cup t' = s$ and $M, t \dashv \varphi$ and $M, t' \dashv \psi$.

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$M, s \Dashv \varphi \vee \psi$ iff $M, s \Dashv \varphi$ and $M, s \Dashv \psi$.

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$M, s \Dashv \varphi \wedge \psi$ iff $\exists t, t' : t \cup t' = s$ and $M, t \Dashv \varphi$ and $M, t' \Dashv \psi$.

Aloni (2022) adds the following atom to make the logic non-classical:

$M, s \models$ NE iff $s \neq \varnothing$.
$M, s \Dashv$ NE iff $s = \varnothing$.

Disjunction in BSML

$M, s \models \varphi \vee \psi$ iff $\exists t, t' : t \cup t' = s \;\&\; M, t \models \varphi \;\&\; M, t' \models \psi$

Pragmatic enrichment: $[\varphi \otimes \psi]^+ = ([\varphi]^+ \otimes [\psi]^+) \wedge \text{NE}$

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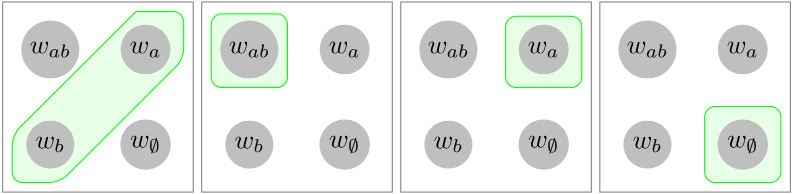

$\models a \vee b$	$\models a \vee b$	$\models a \vee b$	$\not\models a \vee b$
$\models [a \vee b]^+$	$\models [a \vee b]^+$	$\not\models [a \vee b]^+$	$\not\models [a \vee b]^+$

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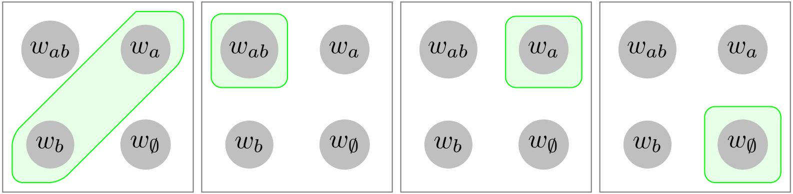

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$\models [a \vee b]^+$	$\models [a \vee b]^+$	$\not\models [a \vee b]^+$	$\not\models [a \vee b]^+$

Note that now formulas denote sets of information states, and not sets of possible worlds.

The proposal

In the logic of information states, agents believe everything that their information states support:

$M, s \models \mathsf{B}\varphi$ iff $M, s \models \varphi$. $\quad$ $M, s \dashv \mathsf{B}\varphi$ iff $\exists t \subseteq s$: $t \neq \varnothing$ & $M, s \dashv \varphi$.

The proposal

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$M, s \models \mathsf{B}\varphi$ iff $M, s \models \varphi$. $M, s \mathrel{=\!\!|} \mathsf{B}\varphi$ iff $\exists t \subseteq s$: $t \neq \varnothing$ & $M, s \mathrel{=\!\!|} \varphi$.

Following Berto (2019), I use Lewis-style set selection function: $f_\varphi : \mathcal{S} \mapsto \mathcal{S}$ such that $f_\varphi(s) = \{v \mid wR_\varphi v \;\&\; w \in s\}$.

The agent believes everything that their φ-accessible information state supports:

$M, s \models \mathsf{B}^\varphi \psi$ iff $M, f_\varphi(s) \models \psi$

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We will use $[\cdot]^+$ to make sure that every part of agent's belief has a non-empty support.

The set selection function

We follow Lewis (1973) and Berto (2019), assuming that each world is associated with a set of spheres: $\S(w) = \{S_0^w, S_1^w, S_2^w \ldots\}$ where $S_0^w = s$ if $i \leq j$ then $S_i^w \subseteq S_j^w$.

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Berto (2019)'s definition: $f_\varphi(w) = \varnothing$ if $|\varphi| = \varnothing$ and otherwise $f_\varphi(w) = S_i^w \cap |\varphi|$ for the smallest S_i^w such that $S_i^w \cap |\varphi| \neq \varnothing$

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But in the logic of information states, formulas denote sets of sets of possible worlds, so we lift this definition:

Let $f_\varphi(s) = \varnothing$, if $|\varphi| = \{\varnothing\}$ or $|\varphi| = \varnothing$ otherwise, let $f_\varphi(s) = max(\{t \mid t \in |\varphi| \;\&\; t \subseteq S_i^s\})$ where S_i^s is the smallest sphere where $\{t \mid t \in |\varphi| \;\&\; t \subseteq S_i^s\} \neq \varnothing$.

Rejection clause

As a rejection clause for belief revision, we will use:

$M, s \mathrel{=\!\!|} \mathsf{B}^{\varphi}\psi$ iff $\exists t \subseteq f_{\varphi}(s)$: $t \neq \varnothing$ & $M, t \mathrel{=\!\!|} \psi$

Note that if $M, s \models \mathsf{B}^{\varphi}\psi$ then $M, s \not\mathrel{=\!\!|} \mathsf{B}^{\varphi}(\psi \vee \chi)$

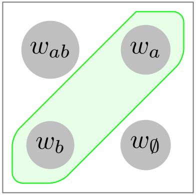
Predictions

Accounting for problematic inferences

<u>(Poss) $\text{B}^{\varphi}(\alpha \vee \beta) \models \langle \text{B} \rangle^{\varphi} \alpha$</u> (Taut2) $\not\models \text{B}^{\varphi}(\varphi \vee \neg\varphi)$ (BSDA) $\text{B}^{\varphi \vee \psi}\alpha \models \text{B}^{\varphi}\alpha$

We use pragmatic enrichment to ensure non-emptiness of every part of the belief:

Suppose $M, s \models \text{B}^{\varphi}[(\alpha \vee \beta)]^{+}$ then $M, f_{\varphi}(s) \models [(\alpha \vee \beta)]^{+}$ hence $\exists t \subseteq f_{\varphi}(s)$: $M, t \models \alpha \wedge \text{NE}$. Hence $M, s \models \langle \text{B} \rangle^{\varphi} \alpha$.



$f_{\varphi}(s)$

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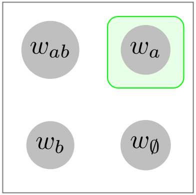

$\models a \vee b$

$\not\models [a \vee b]^+$

Observe that there is a sense in which the agent 'believes' the disjunction. But the belief ascription of the disjunction will fail pragmatically.

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Again we assume pragmatic enrichment: $\text{B}^{[\varphi \vee \psi]^+} \alpha \models \text{B}^{\varphi} \alpha$ We know that: $f_{[\varphi \vee \psi]^+}$ contains the closest witnesses for both φ and ψ and that their set satisfies α:

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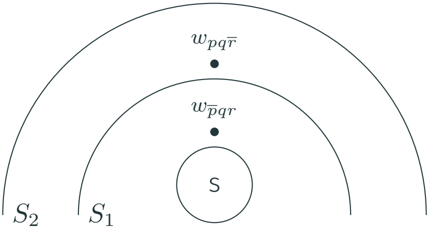

Classical counterexample for $\textsc{b}^{[p\vee q]^+}r$: Observe that S_1 does not contain a p-world. Hence, to support the antecedent, we need to move to S_2. But since S_2 contains a non-r-world #.

Comparison to Berto (2019)

Inference patterns

? **(Success)** $\models \mathsf{B}^{\varphi}\varphi$

✓(Simplif) $\mathsf{B}^{\varphi}(\psi \wedge \chi) \models \mathsf{B}^{\varphi}\psi$

✓(Adjunction)
$\{\mathsf{B}^{\varphi}\psi, \mathsf{B}^{\varphi}\chi\} \models \mathsf{B}^{\varphi}(\psi \wedge \chi)$

✓(Non-monotonicity)
$\mathsf{B}^{\varphi}\psi \not\models \mathsf{B}^{\varphi \wedge \chi}\psi$

✓**(Disjunction 1)** $\mathsf{B}^{\varphi}\psi \not\models \mathsf{B}^{\varphi}(\psi \vee \chi)$

✓(Disjunction 2) $\mathsf{B}^{\varphi}(\psi \vee \chi) \not\models \mathsf{B}^{\varphi}\psi$

✓(LLE/Hyper)
$\{\mathsf{B}^{\varphi}\chi, \varphi \equiv \psi\} \not\models \mathsf{B}^{\psi}\chi$

✗(Incons) $\not\models \mathsf{B}^{\varphi \wedge \neg\varphi}\psi$

✓**(Taut)** $\not\models \mathsf{B}^{\varphi}(\psi \vee \neg\psi)$

✓(PEP) $\{\mathsf{B}^{\varphi}\psi, \mathsf{B}^{\psi}\varphi, \mathsf{B}^{\varphi}\chi\} \models \mathsf{B}^{\psi}\chi$

✓(CUT) $\{\mathsf{B}^{\varphi}\psi, \mathsf{B}^{\varphi \wedge \psi}\chi\} \models \mathsf{B}^{\varphi}\chi$

✓(CM) $\{\mathsf{B}^{\varphi}\psi, \mathsf{B}^{\varphi}\chi\} \models \mathsf{B}^{\varphi \wedge \psi}\chi$

On contradictions

Observe that $M, \varnothing \models p \wedge \neg p$ hence $|p \wedge \neg p| = \{\varnothing\} = |\neg \textsf{NE}|$.

However $M, \varnothing \not\models p \wedge \neg p \wedge \textsf{NE}$ hence $|p \wedge \neg p \wedge \textsf{NE}| = \varnothing = |\textsf{NE} \wedge \neg \textsf{NE}|$.

Weak contradictions are ‘believable’ in an empty (inconsistent) information state.

Strong contradictions are not believable: no state supports them. Belief ascriptions of strong contradictions are not assertible:

$\not\models \textsf{B}^{[\varphi \wedge \neg \varphi]^+} \psi$ and $\not\Dashv \textsf{B}^{[\varphi \wedge \neg \varphi]^+} \psi$

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Success ($\models \text{B}^{\varphi} \varphi$) holds for all *believable* formulas.

Contradictory updates

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Berto suggests using impossible worlds to address this issue. A similar solution is possible here, and it will guarantee the (Incons) principle to hold.

Hyperintensionality

- In the current system $\{\mathsf{B}^{\varphi}\chi, \varphi \equiv \psi\} \not\models \mathsf{B}^{\psi}\chi$. If we keep fixed that '$\equiv$' refers to the truth at the same possible worlds.
- However, if one like Ciardelli et al. (2018) or Aloni (2022) argues that the base logic is also state-based: and $\equiv$ is defined as identity over sets of information states, then the principle ceases to hold.
- In a formal sense, my system is hyperintensional as it defines a logic more fine-grained than classical logic, but it expresses the meaning of formulas using only (sets of) possible worlds.

Logic of Rationality or Pragmatics

- Proponents of hyperintensional theories use linguistic intuitions about belief ascription to motivate various claims.
- They are frequently challenged by classical theorists who argue for a more ‘rational’ closure of the belief set.
- BSML provides a natural distinction between the rational closure of the belief set (in our case: classical logic) and derived from that closure pragma-semantics of belief ascription (via enrichment and NE).
- If the distinction is too strong here, I at least make a case of arguing about it on the basis of the logic of information states.

Conclusions

1. I provided a formal system modelling beliefs and belief revision, which can account for:
    - Intuitions about believing disjunctions
    - Intuitions about learning disjunctive information.
2. This system expresses the meaning of formulas using only (sets of) possible worlds.
3. The system captures at the same time the rational (classical) belief revision and its ‘hyperintensional’ (pragmatic) version.

Thank you!

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